Optimal design of multi-winding transformer using combined FEM, Taguchi and stochastic-deterministic approach

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Abstract: A multi-objective design optimisation for double primary and secondary winding transformers is presented in the study, with the aim to minimise the deviation between the prescribed and designed short-circuit impedance values. The optimisation process involves the exploitation of the results of an efficient and experimentally validated finite element model for leakage field prediction by the Taguchi method. The model is used in conjunction with evolutionary stochastic and deterministic optimisation algorithms in order to derive the optimal winding configuration that meets the technical requirements for the compound multi-winding short-circuit impedance target value.

Nomenclature

- $B$: Magnetic induction (Tesla)
- $\text{BLD}_{HV}$: Width of HV$_1$ (HV$_2$) winding (Fig. 2) (mm)
- $\text{BLD}_{LV}$: Width of LV$_1$ (LV$_2$) winding (Fig. 2) (mm)
- $\text{CCEE} + D_{LV – C}$: Distance between LV$_1$ (LV$_2$) winding and core along y-axis (Fig. 2) (mm)
- $dU$: Average value of percentage deviations of each short-circuit impedance component from the specified value
- $dU_1$, $dU_2$, $dU_3$: Percentage deviation of each short-circuit impedance component from the specified value
- $E_u$: Width of core leg (Fig. 2) (mm)
- $f$: Operating frequency (Hz)
- $F$: Objective function
- $F_{\text{FEM}}$: Objective function computed by FEM
- $F_{\text{fitted}}$: Fitted value of $F$
- $F_{\text{mean}}$: Mean value of $F$
- $F_{\text{standard}}$: Standard deviation of $F$
- $F_2$: Width of small core window (Fig. 2) (mm)
- $G$: Height of core window (Fig. 2) (mm)
- $H$: Magnetic field density (A/m)
- HV$_1$: Upper high-voltage winding
- HV$_2$: Lower high-voltage winding
- $I_{HV1}$: HV$_1$ winding current (A)
- $I_{HV2}$: HV$_2$ winding current (A)
- $I_{HV – HV}$: Distance between HV$_1$ (HV$_2$) windings of two phases (Fig. 2) (mm)
- $I_{LV – C}$: Distance between LV$_1$ (LV$_2$) winding and core along x-axis (Fig. 2) (mm)
- $I_{LV1}$: LV$_1$ winding current (A)
- $I_{LV2}$: LV$_2$ winding current (A)
- $I_{RV – LV}$: Resistive voltage drop of HV$_1$, HV$_2$, LV$_1$ and LV$_2$ windings (%)
- $I_{RV – LV1}$: Resistive voltage drop of HV$_1$, HV$_2$ and LV$_1$ windings (%)
- $I_{RV – LV2}$: Resistive voltage drop of HV$_1$, HV$_2$ and LV$_2$ windings (%)
- $I_{LV1 – LV2}$: Resistive voltage drop of LV$_1$ and LV$_2$ windings (%)
- $I_{IX – HV}$: Inductive voltage drop of HV$_1$, HV$_2$, LV$_1$ and LV$_2$ windings (%)
- $I_{IX – HV1}$: Inductive voltage drop of HV$_1$, HV$_2$ and LV$_1$ windings (%)
- $I_{IX – HV2}$: Inductive voltage drop of HV$_1$, HV$_2$ and LV$_2$ windings (%)
- $I_{IX – LV1}$: Inductive voltage drop of LV$_1$ and LV$_2$ windings (%)
- $J_x$, $J_y$, $J_z$: $x$, $y$, $z$ components of winding current density (A/mm$^2$)
- $K$: Fictitious magnetic field distribution
- $L_{HV1}$: Self-leakage inductance of HV$_1$ winding (H)
\( L_{HV-HV} \)
Mutual leakage inductance between HV1 and HV2 winding (H)

\( L_{HV} \)
Self-leakage inductance of HV2 winding (H)

\( L_{LV-HV} \)
Mutual leakage inductance between LV1 and HV1 winding (H)

\( L_{LV-HV} \)
Mutual leakage inductance between LV1 and HV2 winding (H)

\( L_{LV-LV} \)
Mutual leakage inductance between LV2 and HV1 winding (H)

\( L_{LV-LV} \)
Mutual leakage inductance between LV2 and HV2 winding (H)

\( L_{HV-LV} \)
Total leakage inductance when HV1, HV2, LV1 and LV2 operate under nominal current (H)

\( L_{HV-LV} \)
Total leakage inductance when HV1, HV2, LV1 and LV2 operate under nominal current and HV1 and HV2 are open-circuited (H)

\( L_{LV-LV} \)
Total leakage inductance when LV1 and LV2 operate under nominal current and LV1 is open-circuited (H)

\( L_{LV-LV} \)
Total leakage inductance when LV1 and LV2 operate under nominal current and HV1 and HV2 are open-circuited (H)

\( W_{LV_1} \)
Width of the LV1 winding sub-region of \( \Omega_3 \) (Fig. 6) (mm)

\( W_{LV_2} \)
Width of the LV2 winding sub-region of \( \Omega_3 \) (mm)

\( W_m \)
Magnetic energy of FEM model (J)

\( W_{tetrahedron} \)
Volume of tetrahedral mesh element

\( \alpha_c \)
x-coordinate of the winding centre (mm)

\( \xi_{HV-MIN} \)
Boundaries of the HV1 winding area along \( x \)-axis inside the small core window (Fig. 9) (mm)

\( \xi_{LV-MIN} \)
Boundaries of the LV1 winding area along \( x \)-axis inside the small core window (Fig. 6) (mm)

\( \xi_{HV-MAX} \)
Boundaries of the HV1 winding area along \( x \)-axis inside the large core window (Fig. 9) (mm)

\( \xi_{LV-MAX} \)
Boundaries of the LV1 winding area along \( x \)-axis inside the large core window (mm)

\( X_{LV-MIN} \)
Boundaries of the LV1 winding area along \( x \)-axis inside the large core window (mm)

\( X_{LV-MAX} \)
Boundaries of the LV1 winding area along \( x \)-axis inside the large core window (mm)

\( x_1, x_2, x_3 \)
Components of the design vector (mm)

\( \eta_{HV-MIN} \)
Boundaries of the HV1 winding area along \( y \)-axis (Fig. 9) (mm)

\( \eta_{LV-MIN} \)
Boundaries of the LV1 winding area along \( y \)-axis (mm)

\( \eta_{HV-MAX} \)
Boundaries of the HV1 winding area along \( y \)-axis (mm)

\( \eta_{LV-MAX} \)
Boundaries of the LV1 winding area along \( y \)-axis (mm)

\( Z_{HV-MIN} \)
Boundaries of the HV1 winding area along \( z \)-axis (mm)

\( Z_{HV-MAX} \)
Boundaries of the HV1 winding area along \( z \)-axis (mm)

\( Z_{LV-MIN} \)
Boundaries of the LV1 winding area along \( z \)-axis (mm)

\( Z_{LV-MAX} \)
Boundaries of the LV1 winding area along \( z \)-axis (mm)

\( \xi_{HV-MIN} \)
Boundaries of the HV1 winding area along \( x \)-axis inside the large core window (mm)

\( \xi_{LV-MIN} \)
Boundaries of the LV1 winding area along \( x \)-axis inside the large core window (mm)

\( \xi_{HV-MAX} \)
Boundaries of the HV1 winding area along \( x \)-axis inside the large core window (mm)

\( \xi_{LV-MAX} \)
Boundaries of the LV1 winding area along \( x \)-axis inside the large core window (mm)

\( \xi_{HV-MIN} \)
Boundaries of the HV1 winding area along \( x \)-axis inside the small core window (Fig. 5) (mm)

\( \xi_{LV-MIN} \)
Boundaries of the LV1 winding area along \( x \)-axis inside the small core window (Fig. 5) (mm)

\( \xi_{HV-MAX} \)
Boundaries of the HV1 winding area along \( x \)-axis inside the large core window (mm)

\( \xi_{LV-MAX} \)
Boundaries of the LV1 winding area along \( x \)-axis inside the large core window (mm)

\( \eta_{LV-MIN} \)
Boundaries of the LV1 winding area along \( y \)-axis (Fig. 5) (mm)

\( \eta_{LV-MAX} \)
Boundaries of the LV1 winding area along \( y \)-axis (mm)

\( Z_{HV-MIN} \)
Boundaries of the HV1 winding area along \( z \)-axis (mm)

\( Z_{HV-MAX} \)
Boundaries of the HV1 winding area along \( z \)-axis (mm)

Greek symbols

\( (e_1) \)
Outer ellipse (boundary) of region \( \Omega_5 \) (Fig. 5)

\( (e_1') \)
Outer ellipse (boundary) of region \( \Omega_6 \) (Fig. 5)

\( (e_2) \)
Inner ellipse (boundary) of region \( \Omega_5 \) (Fig. 5)

\( (e_2') \)
Inner ellipse (boundary) of region \( \Omega_6 \) (Fig. 5)

\( \Phi \)
Magnetic scalar potential

\( \phi_{HV_1} \)
Leakage flux of HV1 winding (Wb)

\( \phi_{HV_2} \)
Leakage flux of HV2 winding (Wb)

\( \phi_{LV_1} \)
Leakage flux of LV1 winding (Wb)

\( \phi_{LV_2} \)
Leakage flux of LV2 winding (Wb)

\( \phi_{LV_1-HV_2} \)
Leakage flux between HV1 and HV2 winding (Wb)

\( \phi_{LV_1-HV_1} \)
Leakage flux between LV1 and HV1 winding (Wb)
1 Introduction

Four winding transformer models have been developed in the literature, mostly devoted to transformers with one primary and three secondary windings, involving mainly analytical equations. In [1], a physical γ-based electrical model of a high-voltage transformer with one primary and three secondary windings is presented, along with a method to calculate the leakage inductance for non-uniformly spaced windings, based on analytical equations. In [2], an equivalent circuit model for multiple winding transformers (with one primary and \(n\) secondary windings) is proposed in which the respective winding leakage inductances themselves are treated as coupled inductors with effective coupling coefficients. Reference [3] presents an equivalent-circuit model that accounts for ac winding and core effects in transformers with any number of windings, whereas in [4], a multiple-winding transformer model where each parameter corresponds to a physical magnetic flux is presented. Three-dimensional finite element analysis of multiple-winding transformers is not encountered in the technical literature. It is also observed that multi-winding transformer analysis is in general an issue that is scarcely addressed in transformer design research [5].

A significant difficulty in the analytical determination of the equivalent circuit parameters relies in the fact that they are dependent from the main flux that links all the transformer windings, the self-leakage inductance of each one of them, as well as the four mutual inductances between them [6]. On the contrary, numerical field analysis techniques as the Finite Element Method (FEM) are indicated for the accurate prediction of the interwinding leakage field and the derivation of these parameters in such composite-winding structures. In the case of the double primary and secondary winding transformers examined in the study, four short-circuit impedance values are measured, and are subject to constraints imposed by technical specifications [7]. These values are influenced by the distances between the windings, and the simultaneous achievement of the target values is not straightforward, since the distances between the four windings of one phase that ensure the desired value of one component of short-circuit impedance may result to deviation of the other components from the respective prescribed values. Therefore, the search for the optimal configuration of windings becomes a complex task, taking into account variations of various variables.

Design of experiments (DOE) is a systematic approach to investigation of a system or process. A series of structured tests are designed in which planned changes are made to the input variables of a process or system. The effects of these changes on a pre-defined output are then assessed. In the case of the short-circuit impedance prediction of the multi-winding transformer, DOE can be used as a means to explore the effect of crucial design parameters to the impedance value. The three-dimensional (3D) FEM model is used for impedance value calculation for each value of the design parameters, by conducting a series of ‘numerical experiments’. This combination produces an analytical description of the correlation between the design variables and the short-circuit impedance that can be used as input to various optimisation algorithms.

The present study introduces the coupling of numerical field analysis results with stochastic and deterministic optimisation algorithms to the multi-winding transformer design optimisation. The results of an efficient FEM model are properly exploited by the application of DOE method and provided as input to a combined stochastic-deterministic optimisation scheme. The analysis yields the optimal winding configuration, so as to achieve the desired short-circuit impedance values.

Fig. 1  Active part configuration of the examined multi-winding distribution transformer (the coloured components correspond to the one-phase part modelled in FEM)
The novelty of the proposed method relies on the combination of: (i) an FEM model able to predict transformer short-circuit impedance with the least computational effort, (ii) Taguchi method, which is a proper flexible implementation of the well-established method of DOE, in order to derive a robust and convenient optimisation procedure and (iii) three stochastic optimisation algorithms, with increased usage to electromagnetic problems and a deterministic one for the derivation of the final optimum value, in order to ensure the validity of the results in a complex problem as the one investigated in the study. The optimisation results prove that the proposed analysis results to a better optimum value than the one yielded separately by each algorithm and improve significantly the initial transformer design.

2 Description of the multi-winding transformer

The examined 1000 kVA three-phase distribution transformer comprises two independent (primary) high-voltage (HV) and (secondary) low-voltage (LV) windings per phase, wound around the same core leg, depicted as upper and lower windings in Figs. 1 and 2. The upper and lower windings of each phase are identical, in order to produce the same voltage level, however, this is not a general requirement for this kind of transformers. HV$_1$ and HV$_2$ comprise 1210 turns of copper wire whereas LV$_1$ and LV$_2$ comprise 11 turns of copper sheet. The nominal voltage of HV$_1$ and HV$_2$ is 20 000 V while the nominal voltage of LV$_1$ and LV$_2$ is 315 V. Four short-circuit tests are carried out, yielding the short-circuit impedance components $U_{HV - LV}$, $U_{HV - LV_1}$, $U_{LV_1 - LV_2}$ and $U_{LV_1 - LV_2}$ listed in Table 1 and described in the followings:

$U_{HV - LV}$: LV$_1$ and LV$_2$ are short-circuited, HV$_1$ and HV$_2$ operate under nominal current. This test determines the transformer impedance value when all the windings operate under nominal current.

It must be noted that, since LV$_1$ is identical to LV$_2$ and HV$_1$ is identical to HV$_2$, $\Phi_{LV_1} = \Phi_{LV_2}$, $\Phi_{HV_1} = \Phi_{HV_2}$, $\Phi_{LV_1 - HV_1} = \Phi_{LV_2 - HV_2}$ and $\Phi_{LV_1 - HV_2} = \Phi_{LV_2 - HV_1}$, therefore (1) becomes

$$\Phi_{total} = 2\Phi_{LV_1} + 2\Phi_{HV_1} + 2\Phi_{LV_1 - HV_1}$$

$$+ 2\Phi_{LV_2} + 2\Phi_{HV_2} + 2\Phi_{LV_2 - HV_2} + 2\Phi_{LV_1 - HV_2}$$

$$+ 2\Phi_{LV_1 - HV_2} + 2\Phi_{HV_1 - HV_2}$$

$$\Phi_{total} = 2\Phi_{LV_1} + 2\Phi_{LV_2} + 2\Phi_{LV_1 - HV_1} + 2\Phi_{LV_2 - HV_2}$$

$$+ 2\Phi_{LV_1 - LV_2} + 2\Phi_{HV_1 - HV_2}$$

(2)

Table 1 Short-circuit tests

<table>
<thead>
<tr>
<th>Short-circuit tests</th>
<th>LV$_1$</th>
<th>LV$_2$</th>
<th>HV$_1$</th>
<th>HV$_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st test: $U_{HV - LV}$</td>
<td>SC</td>
<td>SC</td>
<td>NC</td>
<td>NC</td>
</tr>
<tr>
<td>2nd test: $U_{HV - LV_1}$</td>
<td>SC</td>
<td>OC</td>
<td>NC</td>
<td>NC</td>
</tr>
<tr>
<td>3rd test: $U_{HV - LV_2}$</td>
<td>OC</td>
<td>SC</td>
<td>NC</td>
<td>NC</td>
</tr>
<tr>
<td>4th test: $U_{LV_1 - LV_2}$</td>
<td>SC</td>
<td>NC</td>
<td>OC</td>
<td>OC</td>
</tr>
</tbody>
</table>

SC, short-circuited; NC, nominal current; OC, open-circuited

In this case, the total leakage flux can be divided into the four self-leakage inductance components of LV$_1$, LV$_2$, HV$_1$ and HV$_2$ and the mutual leakage flux between them, also depicted in Fig. 3.

$$U_{HV - LV_1}: LV_1$$ is short-circuited, $LV_2$ winding is open-circuited and, HV$_1$ and HV$_2$ operate under nominal current. This test determines the transformer impedance
value when the two primary windings (HV1 and HV2) and only one secondary winding (LV1) operate under nominal current.

In this case, the total leakage flux is divided into the three self-leakage inductance components of LV1, HV1 and HV2 and the mutual leakage flux between them, also depicted in Fig. 3.

\[
\Phi_{\text{test}2} = \Phi_{LV1} + \Phi_{HV1} + \Phi_{HV2} + \Phi_{LV1-HV1} + \Phi_{LV1-LV2} + \Phi_{HV1-HV2}
\]

(3)

\(U_{HV-LV2}\): LV1 winding is open-circuited, LV2 winding is short-circuited and HV1 and HV2 operate under nominal current. This test determines the transformer impedance value when the two primary windings (HV1 and HV2) and only one secondary winding (LV2) operate under nominal current.

In this case, the total leakage flux is divided into the three self-leakage inductance components of LV2, HV1 and HV2 and the mutual leakage flux between them, also depicted in Fig. 3.

\[
\Phi_{\text{test}3} = \Phi_{LV2} + \Phi_{HV1} + \Phi_{HV2} + \Phi_{LV1-HV1} + \Phi_{LV2-HV2} + \Phi_{HV1-HV2}
\]

(4)

Since \(\Phi_{LV1} = \Phi_{LV2}\), \(\Phi_{HV1} = \Phi_{HV2}\), \(\Phi_{LV1-HV1} = \Phi_{LV1-HV2}\), and \(\Phi_{LV2-HV1} = \Phi_{LV2-HV2}\), as mentioned above, \(\Phi_{\text{test}} = \Phi_{\text{test}2}\) and \(U_{HV-LV1} = U_{HV-LV2}\).

\(U_{LV1-LV2}\): LV1 winding is short-circuited, LV2 winding operates under nominal current and HV1 and HV2 are open-circuited. This test determines the transformer impedance value when only the two secondary windings (LV1 and LV2) operate under nominal current.

In this case, the total leakage flux is divided into the three self-leakage inductance components of LV1 and LV2 and the
mutual leakage flux between them, also depicted in Fig. 3.

\[ \Phi_{\text{total}}^{\text{test}} = \Phi_{\text{LV}_1} + \Phi_{\text{LV}_2} + \Phi_{\text{LV}_1-LV_2} \quad (5) \]

It must be noted that the above tests do not cover all of the short-circuit components of the multi-winding transformer, that is there are also components \( U_{\text{HV}_1-LV} \), \( U_{\text{HV}_2-LV} \) and \( U_{\text{HV}_1-HV_2} \). These components are not measured by the manufacturer, since they do not correspond to possible operating conditions for the examined transformer (i.e. operation with one primary winding and two secondary windings or operation of two primary windings only) so the analysis of the study is focused on the four components presented above.

3 Transformer FEM model

3.1 Model configuration

As simulation of product performance using FEM invariably takes considerable computing resources and time, the major challenge encountered in FEM-based optimisation process for a practical problem is that the designer can only perform a limited number of design trials before reaching the optimal [8]. For this purpose, increase in the efficiency of the FEM model is crucial, especially when combined to optimisation algorithms. The main objective in the development of the FEM model presented in the study was therefore to achieve the maximum accuracy at the minimum computational cost.

For the accurate calculation of the interwinding leakage field, an efficient transformer FEM model has been developed, based on a particular scalar potential formulation, enabling the 3D magnetostatic field analysis [9, 10]. The model is extended to the analysis of the leakage field of distribution transformers with double primary and secondary windings and it is particularly suitable for use with optimisation algorithms, as it reduces the total time needed for the magnetic field calculation during each iteration. Special consideration is given to the detailed winding geometry, taking into account the elliptic shape of the winding corners and the cooling ducts dimensions [10]. The FEM model (Fig. 4) comprises the upper and lower LV and HV windings of one phase, as well as the small and large iron core that surrounds them. An air box, whose dimensions are equal to the transformer tank dimensions, surrounds the active part, therefore confining the field calculation to this domain. Owing to the symmetries of the problem, the solution domain is reduced to one half of the device (i.e. only half of the transformer width is modelled and the \( xz \)-plane crosses the active part symmetry plane). These symmetries were taken into account by the imposition of Neumann boundary condition (\( \frac{\partial \Phi}{\partial n} = 0 \)) along \( xz \)-plane and the three outer faces of the air box.

3.2 Magnetic scalar potential formulation

The model is based on a particular scalar potential formulation, enabling the 3D magnetostatic field analysis. According to this method, the magnetic field strength \( H \) is conveniently partitioned to a rotational and an irrotational part as follows

\[ H = K - \nabla \Phi \quad (6) \]

where \( \Phi \) is a scalar potential extended all over the solution domain, while \( K \) is a vector quantity (fictitious field distribution), that satisfies the following conditions [9]:

1. \( K \) is limited in a simply connected sub-domain comprising the conductor;
2. \( \nabla \times K = J \), in the conductor and \( \nabla \times K = 0 \) outside it;
3. \( K \) is perpendicular on the sub-domain boundary.

The above formulation satisfies Ampere’s law for an arbitrary contour in the sub-domain.

The representation of the magnetic field sources, that is the windings current in the case of the transformer magnetic field, is carried out with the use of a fictitious field distribution \( K \), which must satisfy the conditions above. For the calculation of \( K \), a simply connected sub-domain must be defined for each winding, comprising its conductors. Fig. 5 shows the bottom view of the sub-domain corresponding to the LV₁ winding of Fig. 1.

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**Fig. 4** FEM mesh of the transformer active part

- **a** Low mesh density
- **b** High mesh density

**Fig. 5** Regions (top view of the winding) of the sub-domain used in the calculation of the fictitious field distribution \( K_z \) corresponding to the LV₁ winding of Fig. 1
winding of Fig. 1. This sub-domain is divided into six regions \((\Omega_1, \Omega_2, \Omega_3, \Omega_4, \Omega_5, \Omega_6)\), in order to facilitate the calculation. The symbols shown in Fig. 5 are described in the followings:

\[
X_{LV,MIN}, X_{LV,MAX}: \text{boundaries of the LV}_1 \text{winding area along x-axis inside the small-core window};
\]

\[
X'_{LV,MIN}, X'_{LV,MAX}: \text{boundaries of the LV}_1 \text{winding area along x-axis inside the large-core window};
\]

\[
Y_{LV,MIN}, Y_{LV,MAX}: \text{boundaries of the LV}_1 \text{winding area along y-axis};
\]

\[
J_x, J_y, J_z: x, y \text{ components of winding current density};
\]

\[
X_C: \text{x-coordinate of the winding centre}.
\]

The calculation of \(K\) for \(\Omega_1\) and \(\Omega_2\) is quite straightforward, given the winding dimensions along \(x, y\)- and \(z\)-axis:

1. **Region \(\Omega_2\):** In this region, \(J_y = J_z = 0\). The current density \(J_x\) is given by

\[
J_x = N_{LV_1}I_{LV_1} \times \frac{Z}{X_{LV,MAX} - X_{LV,MIN}}
\]

where \(N_{LV_1}\) and \(I_{LV_1}\) are the turns and current of LV_1 winding, respectively.

The distribution \(K\) must be perpendicular to \(\Omega_1\) boundary (third condition described previously). Therefore, it consists of component \(K_z\) only, while \(K_x = K_y = 0\). The second condition described previously yields

\[
\nabla \times \mathbf{K} = \mathbf{J} \Rightarrow K_z = -\int_{\Omega_1} J_x \, dx \Rightarrow K_z = N_{LV_1}I_{LV_1}Z \times \frac{X_{LV,MIN} - X}{X_{LV,MIN} - X_{LV,MAX}}
\]

2. **Region \(\Omega_3\):** In this region, \(J_y = J_z = 0\) and

\[
J_x = -N_{LV_1}I_{LV_1} \times \frac{Z}{X_{LV,MAX} - X_{LV,MIN}}
\]

while \(K_z\) derives from

\[
K_z = -\int_{\Omega_3} J_x \, dx = N_{LV_1}I_{LV_1}Z \times \frac{X_{LV,MAX} - X}{X_{LV,MAX} - X_{LV,MIN}}
\]

3. **Region \(\Omega_5\):** In this region, \(J_y = J_z = 0\).

With respect to Fig. 5, In the case of the LV_1 winding, the four ducts are placed symmetrically in the centre of region \(\Omega_3\), dividing the region into nine sub-regions: since the width of each ‘duct’ sub-region is equal to \(W_{DUCT}\), the width of remaining five ‘winding’ sub-regions is equal to the width of the LV_1 winding inside the core windows, divided into five (i.e. the number of the ‘winding’ sub-regions) of width \(W_{LV_1}\).

The distribution \(K_z\) of the region \(\Omega_3\) corresponding to the LV_1 winding has the form of Fig. 7, which shows the \(K_z\) distribution along a plane parallel to \(y\)-axis, crossing the centre \(X_C\) of the winding.

The equation of distribution \(J_y\) for region \(\Omega_3\) of LV_1 winding (Fig. 5) is given by (11). The first branch of (11) refers to the ‘winding’ sub-regions, whereas the second branch refers to the ‘duct’ sub-regions. (see (11))

\[
W_{LV_1} = \frac{BLD_{LV}}{5}
\]

where \(BLD_{LV}\) is the width of the LV_1 winding inside the core windows (Fig. 2).

Therefore, \(K_z\) is given by (see (13))

4. **Region \(\Omega_4\):** The application of continuity boundary condition for \(K_z\) between regions \(\Omega_4\) and \(\Omega_3\) yields:

\[
K_z(Y = Y_{LV,MIN}_{LV_1}) = K_z(Y = Y_{LV,MIN}_{LV_1})
\]

\[
\Rightarrow K_z(Y = Y_{LV,MIN}_{LV_1}) = N_{LV_1}I_{LV_1}Z
\]

The application of continuity boundary condition between regions \(\Omega_3\) and \(\Omega_4\) or \(\Omega_5\) and \(\Omega_4\) results to the same equation for \(K_z\) in region \(\Omega_4\).

\[
J_y = N_{LV_1}I_{LV_1}Z \left\{ \frac{1}{5W_{LV_1}} [Y + Y_{LV,MIN} + jW_{LV_1} + (j - 1)W_{DUCT}] + \frac{|j - 5|}{5}, \quad j = 1, \ldots, 5 \right.\]

\[
\left. \frac{|j - 5|}{5}, \quad j = 1, \ldots, 4 \right\}
\]

\[
K_z = \int_{\Omega_5} J_y \, dy = N_{LV_1}I_{LV_1}Z \left\{ \frac{1}{5W_{LV_1}} [Y + Y_{LV,MIN} + jW_{LV_1} + (j - 1)W_{DUCT}] + \frac{|j - 5|}{5}, \quad j = 1, \ldots, 5 \right.\]

\[
\left. \frac{|j - 5|}{5}, \quad j = 1, \ldots, 4 \right\}
\]

\[
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\]


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I appearing in denote the derive from the (X, Z′ − 1, X1(17) − BLD1V + Ywinding. The symbols = − 5 of = × V region division in 'ducts' and 'winding' sub-regions (Fig. 5). The Institution of Engineering and Technology 2012 doi: 10.1049/iet-epa.2011.0310 XLV, Y = − 444 − V ... www.ietdl.org X ... IET Electr. Power Appl. ... 437–454 © The Institution of Engineering and Technology 2012
The derivation of distribution $K$ for the HV winding is similar, resulting to equation identical to (21), with the respective boundaries of the winding along $x$, $y$- and $z$-axis, as shown in Fig. 9. (see (22)).

The respective fictitious field distributions for windings LV$_2$ and HV$_2$ are identical to (21) and (22), respectively, with the only difference relying in the fact that index ‘LV’ and ‘HV’ must be replaced by ‘LV$_2$’ and ‘HV$_2$’, respectively.

The representation of current sources through distribution $K$ has the advantage of being compatible with the discrete scheme of first order tetrahedral elements so that it does not suffer from cancellation errors, present in case of using Biot-Savart law to determine source field distribution. This characteristic assures high accuracy at low mesh densities, which is the prominent advantage of the developed FEM model.

### 3.3 Calculation of short-circuit impedance components

#### 3.3.1 $U_{HV-LV}$

The short-circuit impedance is calculated with the use of the magnetic energy of the FEM model $W_m$. The total magnetic energy derives from the magnetic field density ($H$) and the magnetic induction ($B$) in each tetrahedral element of the mesh, by summation of the product $(1/2)BH$ tetrahedron of all the mesh elements (thus integrating the elementary magnetic energy density $(dW_m/dV)$ over the mesh volume).

When the LV$_1$ and LV$_2$ windings are short-circuited and the HV$_1$ and HV$_2$ windings operate under nominal current

$$W_m = \frac{1}{2} (L_{LV_1} I_{LV_1}^2 + L_{LV_2} I_{LV_2}^2 + L_{HV_1} I_{HV_1}^2 + L_{HV_2} I_{HV_2}^2)$$

$$+ L_{LV_1-HV_1} I_{LV_1} I_{HV_1} + L_{LV_1-HV_2} I_{LV_1} I_{HV_2}$$

$$+ L_{LV_2-HV_1} I_{LV_2} I_{HV_1} + L_{LV_2-HV_2} I_{LV_2} I_{HV_2}$$

$$+ L_{LV_1-LV_2} I_{LV_1} I_{LV_2} + L_{HV_1-HV_2} I_{HV_1} I_{HV_2}$$  \hspace{1cm} (23)

Since LV$_1$ is identical to LV$_2$ and HV$_1$ is identical to HV$_2$, $I_{LV_1} = I_{LV_2}$ and $I_{HV_1} = I_{HV_2}$ therefore (23) becomes

$$W_m = \frac{1}{2} (L_{LV_1} I_{LV_1}^2 + L_{LV_2} I_{LV_2}^2 + L_{HV_1} I_{HV_1}^2 + L_{HV_2} I_{HV_2}^2)$$

$$+ L_{LV_1-HV_1} I_{LV_1} I_{HV_1} + L_{LV_1-HV_2} I_{LV_1} I_{HV_2}$$

$$+ L_{LV_2-HV_1} I_{LV_2} I_{HV_1} + L_{LV_2-HV_2} I_{LV_2} I_{HV_2}$$

$$+ L_{LV_1-LV_2} I_{LV_1} I_{LV_2} + L_{HV_1-HV_2} I_{HV_1} I_{HV_2}$$  \hspace{1cm} (24)

In the case of the examined multi-winding transformer

$$\frac{I_{LV_1}}{I_{HV_1}} = \frac{I_{LV_2}}{I_{HV_2}} = \frac{N_{HV_1}}{N_{LV_1}} = \frac{N_{HV_2}}{N_{LV_2}}$$  \hspace{1cm} (25)

$$K_s = \left\{ \begin{array}{ll}
N_{HV_1} I_{HV_1} Z \times \frac{X_{HV,MIN} - X}{X_{HV,MIN} - X_{HV,MAX}}, & \text{region } \Omega_1 \\
N_{HV_1} I_{HV_1} Z \times \frac{X'_{HV,MAX} - X}{X'_{HV,MAX} - X'_{HV,MIN}}, & \text{region } \Omega_2 \\
- \frac{1}{5} W_{HV_1} \left[ Y + Y_{HV,MIN} + j W_{HV_1} + (j - 1) W_{DUCT} \right] + \frac{|j - 5|}{5}, & j = 1, \ldots, 5, \text{ region } \Omega_3 \\
\frac{|j - 5|}{5}, & j = 1, \ldots, 4, \text{ region } \Omega_4 \\
N_{LV_1} I_{HV_1} Z \times \frac{(e_1) - 1}{(e_1) - (e_2)}, & \text{region } \Omega_5 \\
N_{LV_1} I_{HV_1} Z \times \frac{(e_2) - 1}{(e_1) - (e_2)}, & \text{region } \Omega_6
\end{array} \right\}$$  \hspace{1cm} (22)
Combining (24) and (25) we get

\[
W_m = \frac{1}{2} \left( L_{LV_1} I_{LV_1}^2 + L_{LV_2} I_{LV_1}^2 + L_{HV_1} \left( \frac{I_{HV_1}}{N_{HV_1}/N_{LV_1}} \right)^2 \right) + L_{HV_2} \left( \frac{I_{HV_1}}{N_{HV_1}/N_{LV_1}} \right)^2 + L_{LV_1-HV_1} I_{LV_1} I_{HV_1} + L_{LV_2-HV_1} I_{LV_1} I_{HV_1} + L_{LV_1-HV_2} I_{LV_1} I_{HV_2} + L_{HV_1-HV_2} I_{HV_1} I_{HV_2}
\]

becomes

\[
W_m = \frac{1}{2} \left( L_{LV_1} I_{LV_1}^2 + L_{HV_1} \left( \frac{I_{HV_1}}{N_{HV_1}/N_{LV_1}} \right)^2 \right) + L_{LV_1-HV_1} I_{LV_1} I_{HV_1} + L_{LV_1-HV_2} I_{LV_1} I_{HV_2} + L_{HV_1-HV_2} I_{HV_1} I_{HV_2}
\]

Combining (30) and (25) we get

\[
W_m = \frac{1}{2} \left( L_{LV_1} I_{LV_1}^2 + L_{HV_1} \left( \frac{I_{HV_1}}{N_{HV_1}/N_{LV_1}} \right)^2 \right) + L_{HV_2} \left( \frac{I_{HV_1}}{N_{HV_1}/N_{LV_1}} \right)^2 + L_{LV_1-HV_1} I_{LV_1} I_{HV_1} + L_{LV_1-HV_2} I_{LV_1} I_{HV_2} + L_{HV_1-HV_2} I_{HV_1} I_{HV_2}
\]

The inductive voltage drop \( \Delta X_{HV-LV} \) of the windings (referred to winding \( LV_1 \)) will be therefore equal to

\[
\Delta X_{HV-LV} = I_{LV_1} \frac{2 \pi N_{LV_1}^2 L_{HV-LV}}{V_{LV_1}}
\]

where \( L_{HV-LV} \) is the total leakage reactance (H) of the windings when \( LV_1 \), \( LV_2 \), \( HV_1 \) and \( HV_2 \) operate under nominal current and \( f \) is the operating frequency (Hz).

The respective short-circuit impedance will be equal to

\[
U_{HV-LV} = \sqrt{\left( \Delta X_{HV-LV} \right)^2 + \left( IR_{HV-LV} \right)^2}
\]

where \( IR_{HV-LV} \) is the total respective resistive voltage drop of \( LV_1 \), \( LV_2 \), \( HV_1 \) and \( HV_2 \) windings.

### 3.3.2 \( U_{HV-LV} \) and \( U_{HV-LV} \)

When the \( LV_1 \) winding is short-circuited, \( LV_2 \) winding is open-circuited and the \( HV_1 \) and \( HV_2 \) windings operate under nominal current

\[
W_m = \frac{1}{2} \left( L_{LV_1} I_{LV_1}^2 + L_{HV_1} I_{HV_1}^2 + L_{HV_2} I_{HV_2}^2 \right) + L_{LV_1-HV_1} I_{LV_1} I_{HV_1} + L_{LV_1-HV_2} I_{LV_1} I_{HV_2} + L_{HV_1-HV_2} I_{HV_1} I_{HV_2}
\]

Since \( HV_1 \) is identical to \( HV_2 \), \( I_{HV_1} = I_{HV_2} \), therefore (29) becomes

\[
W_m = \frac{1}{2} \left( L_{LV_1} I_{LV_1}^2 + L_{HV_1} I_{HV_1}^2 + L_{HV_2} I_{HV_2}^2 \right) + L_{LV_1-HV_1} I_{LV_1} I_{HV_1} + L_{LV_1-HV_2} I_{LV_1} I_{HV_2} + L_{HV_1-HV_2} I_{HV_1} I_{HV_2}
\]

Combining (30) and (25) we get

\[
W_m = \frac{1}{2} \left( L_{LV_1} I_{LV_1}^2 + L_{HV_1} I_{HV_1}^2 + L_{HV_2} I_{HV_2}^2 \right) + L_{LV_1-HV_1} I_{LV_1} I_{HV_1} + L_{LV_1-HV_2} I_{LV_1} I_{HV_2} + L_{HV_1-HV_2} I_{HV_1} I_{HV_2}
\]
current, we get

\[
\frac{W_m}{(N_{LV_1}I_{LV_1})^2} = \frac{1}{2} \left( \frac{L_{LV_2}}{N_{LV_2}^2} + \frac{L_{HV_1}}{N_{HV_1}^2} + \frac{L_{HV_2}}{N_{HV_2}^2} \right) + \frac{L_{LV_1-LV_2}}{N_{HV_1}N_{LV_2}} + \frac{L_{LV_2-HV_2}}{N_{HV_1}N_{LV_2}} + \frac{L_{HV_1-HV_2}}{N_{HV_1}N_{LV_2}}
\]

\[
= L_{HV-LV_2}^{\text{total}}
\]

(34)

\[
IX_{HV-LV_2}(\%) = \frac{I_{LV_2} \cdot 2 \cdot \pi \cdot f \cdot N_{LV_2}^2 \cdot L_{HV-LV_2}^{\text{total}}}{V_{LV_2}}
\]

(35)

\[
U_{HV-LV_1}(\%) = \sqrt{(IX_{HV-LV_2})^2 + (IR_{HV-LV_2})^2}
\]

(36)

Since windings LV_1 and LV_2 are identical (with same nominal voltage, current and turns), the comparison of (31)–(33) and (34)–(36) yields

\[
U_{HV-LV_1}^{\text{computed}} = U_{HV-LV_2}^{\text{computed}}
\]

(37)

3.3.3 \(U_{LV_1-LV_2}\): When the LV_1 winding is short-circuited and LV_2 winding operates under nominal current while HV_1 and HV_2 windings are open-circuited

\[
W_m = W_m^{\text{total}}
\]

\[
= \frac{1}{2} (L_{LV_1}^2I_{LV_1}^2 + L_{LV_2}^2I_{LV_2}^2) + L_{LV_1-LV_2}^2I_{LV_1}I_{LV_2}
\]

(38)

Since LV_1 is identical to LV_2, \(I_{LV_1} = I_{LV_2}\) and (38) becomes

\[
W_m = \frac{1}{2} (L_{LV_1}^2I_{LV_1}^2 + L_{LV_2}^2I_{LV_2}^2) + L_{LV_1-LV_2}^2I_{LV_1}^2
\]

(39)

and

\[
\frac{W_m}{(I_{LV_2})^2} = \frac{1}{2} L_{LV_1} + \frac{1}{2} L_{LV_2} + L_{LV_1-LV_2}
\]

(40)

The inductive voltage drop \(IX_{LV_1-LV_2}\) of the windings (referred to winding LV_1) will be therefore equal to

\[
IX_{LV_1-LV_2}(\%) = \frac{I_{LV_1} \cdot 2 \cdot \pi \cdot f \cdot L_{LV_1}^{\text{total}}}{V_{LV_1}}
\]

(41)

where \(L_{LV_1-LV_2}^{\text{total}}\) is the total leakage inductance (H) of the windings when the LV_1 and LV_2 operate under nominal current while HV_1 and HV_2 windings are open-circuited and \(f\) is the operating frequency (in Hz).

The respective short-circuit impedance will be equal to

\[
U_{LV_1-LV_2}(\%) = \sqrt{(IX_{LV_1-LV_2})^2 + (IR_{LV_1-LV_2})^2}
\]

(42)

where \(IR_{LV_1-LV_2}\) is the total respective resistive voltage drop of LV_1 and LV_2 windings.

### 3.4 Mesh construction

The construction of the transformer 3D FEM mesh has derived through proper discretisation in winding areas with special interest, resulting to the selection of the most efficient in terms of computational requirements and accuracy [10]. For the construction of the mesh appearing in Fig. 4 first order tetrahedral elements were used. Although the use of higher order elements was possible, it was not implemented in the pre-processor, as they did not contribute to the increase of the model accuracy, while they resulted to greater computation time. As the construction of the mesh is crucial for the accuracy of the calculations conducted by the finite element method, careful consideration was given on its density and homogeneity. Moreover, as the computation time increases with the number of mesh nodes, the total mesh size should be confined below a number that guarantees a not excessively time consuming solution model. For this purpose, meshes of various densities were constructed, after refinement in areas of special interest: that is why the nodes density in the mesh of Fig. 4 is greater in the windings area, in order to obtain greater accuracy in the magnetic field sources region. Two mesh densities were finally selected: a coarse mesh, of about 5000 nodes (Fig. 4a) and a mesh of higher density (Fig. 4b) of approximately 30 000 nodes. Although the mesh of Fig. 4a is able to produce accurate results with very low execution time, as described in Section 3.5, the mesh of Fig. 4b is also used in the Taguchi analysis with noise factors (as explained in Section 4) in order to reduce potential errors because of the finite element size.

### 3.5 Validation by measurements

In order to validate the FEM model, its results were compared with the actual values measured on the multi-winding transformer described in Section 2. The distances between the windings (Fig. 2) in this transformer are \(x_1 = 12\) mm, \(x_2 = 4\) mm and \(x_3 = 8\) mm.

**Table 2** lists the computed short-circuit impedance values and the respective measured values for the four short-circuit tests described in Section 2.1, as well as their deviation, indicating the good accuracy of the FEM model, since this deviation does not exceed 3%. **Table 2** also includes the specified values of the short-circuit impedances (the deviation of the actual short-circuit impedance and the specified values must not exceed 10%), along with the respective deviation between the computed and specified values. It must also be noted that all results are obtained by a relatively coarse mesh (mesh of Fig. 4a), with total execution time less than 1 min in a personal computer of medium computational capability.

It is known that 2–3% is in general the limit of accuracy provided by standard FEM models. However, this accuracy

<table>
<thead>
<tr>
<th>Short-circuit impedance</th>
<th>Computed by FEM, %</th>
<th>Measured, %</th>
<th>Specified, %</th>
<th>Deviation between computed and measured, %</th>
<th>Deviation between computed and specified, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>(U_{HV-LV})</td>
<td>7.54</td>
<td>7.67</td>
<td>7.20</td>
<td>1.69</td>
<td>4.74</td>
</tr>
<tr>
<td>(U_{HV-LV_1} = U_{HV-LV_2})</td>
<td>6.35</td>
<td>6.12</td>
<td>6.00</td>
<td>2.00</td>
<td>5.82</td>
</tr>
<tr>
<td>(U_{LV_1-LV_2})</td>
<td>9.88</td>
<td>10.28</td>
<td>10.00</td>
<td>2.80</td>
<td>1.20</td>
</tr>
</tbody>
</table>

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is obtained at high mesh densities (several thousands of nodes in the case of 3D meshes) requiring significant execution time requirements. In the case of the developed FEM model, this accuracy is obtained at a low mesh density, which is its major advantage. This is achieved mainly by the adoption of the particular scalar potential formulation described in Section 3.2, as well as the optimised mesh construction described in Section 3.4.

4 Geometry optimisation of windings

4.1 Mathematical formulation

The interwinding leakage field is mainly influenced by the distance between the lower and upper windings (vertical gap) and the inner and outer windings (horizontal gap). These gaps, also depicted in Fig. 2, are the design variables of the geometry optimisation problem. The goal is to achieve the optimum balance between these values, in order to minimise the difference between the specified and designed short-circuit impedances. The analytical expression of the objective function is given by

$$F = \sqrt{(U_1 - U_{1\text{spec}}^2)^2 + (U_2 - U_{2\text{spec}}^2)^2 + (U_3 - U_{3\text{spec}})^2}$$  (43)

where $U_1 = U_{HV-LV}$, $U_2 = U_{LV-LV}$, and $U_3 = U_{LV-HV}$, while $U_{1\text{spec}}$, $U_{2\text{spec}}$, and $U_{3\text{spec}}$ are the respective specified values. The upper bounds of $x_1^\text{max}$, $x_2^\text{max}$, and $x_3^\text{max}$, imposed by the geometrical restrictions of the active part (Fig. 2), are defined by the following equations

$$x_1^\text{max} = G = (2TD_{LV} + 2\text{CCEE} + 2D_{LV-C})$$  (44)

$$x_2^\text{max} = 0.5 \cdot (TD_{LV} - TD_{HV})$$  (45)

$$x_3^\text{max} = 0.5 \cdot F2$$

$$- (BLD_{HV} + BLD_{LV} + I_{HV-HV} + I_{LV-C})$$  (46)

The lower bound $x_1^\text{min}$ of $x_1$ (minimum distance between LV and LV winding) is imposed by the basic insulation level of LV windings (BIL$_{LV}$). For nominal voltage lower than 11000 V, BIL$_{LV}$ must be equal to 10 kV and the minimum distance between LV and LV winding is equal to 1.5 mm. The lower bound $x_3^\text{min}$ of $x_3$ (minimum distance between HV$1$ and LV$1$ winding, which is equal to the distance between HV$2$ and LV$2$ winding) is imposed by the basic insulation level of HV windings (BIL$_{HV}$). For nominal voltage between 17.5 and 24 kV, BIL$_{HV}$ must be equal to 125 kV and the minimum distance between HV$1$ and LV$1$ winding is equal to 6.9 mm. No lower bound is applied to $x_2$: however, since the distance between HV$1$ and HV$2$ winding is equal to $x_1 + 2x_2$ and BIL$_{HV}$ is equal to 125 kV, the minimum value of $x_1 + 2x_2$ must be equal to 6.9 mm.

4.2 Design of experiments

DOE is used for the process of planning, designing and conducting a series of calculations based on the FEM model so that valid conclusions can be drawn effectively for the objective short-circuit impedance values. Numerical experiments are conducted in order to approach the relationship between the response value (namely, the short-circuit impedance) and the design variables [11].

The response function is used as input to various optimisation algorithms for the calculation of the optimal winding geometry.

4.2.1 Taguchi orthogonal array factorial design with noise factors: Instead of full factorial designs, the Taguchi orthogonal array (OA) design is used in the optimisation process. In this method, an orthogonal array that depends on the number of factors and levels is used to study the parameters’ variation effect [12, 13]. This approach eliminates significantly the number of experiments used as input in DOE. In our case, where five different levels are considered for each one of the three design variables, a full factorial design would require $5^3 = 125$ values for $U_1$, $U_2$, and $U_3$, yielded by the FEM model. However the Taguchi method requires only 25 experiments. In our case, FEM is used for the derivation of these experiments, so they may be considered as numerical experiments. Table 3 lists the values of $U_1$, $U_2$, $U_3$ and $F_{\text{FEM}}$ for these numerical experiments, obtained by the FEM model of Section 3 with the use of a coarse mesh (Fig. 4a). The five levels of design variables $x_1$, $x_2$ and $x_3$ are [8 9 10 11 12], [3 4 5 6 7] and [6 7 8 9 10], respectively, while $U_{1\text{spec}} = 7.2\%$, $U_{2\text{spec}} = 6\%$, $U_{3\text{spec}} = 10\%$. Since the mean time for each FEM execution is equal to 1 min, as reported in Section 3, the overall time for the derivation of the three short-circuit impedance components of the 25 experiments of Table 2 was equal to 3.25·1 = 75 min.

In order to increase the robustness of the method, noise factors are inserted in the analysis, that is factors of the experiments that cannot be easily controlled. In our case, since the numerical experiments are carried out with the use of the FEM model of Section 3, noise factors are relevant
to the finite element mesh, which influences the accuracy of calculations. For this purpose, the numerical experiments of Table 3 have been repeated with the use of mesh of higher density (Fig. 4b), and the respective results are listed in Table 4. The execution time in this case increases to 10 min per each execution, resulting to an overall time equal to 3.25·10 = 750 min.

4.2.2 Analysis of variances and analysis of mean: Analysis of mean (ANOM) helps identify the optimal factor combinations whereas analysis of variances (ANOVA) establishes the relative significance of factors in terms of their contribution to the objective function. More specifically, the plots of main factor effects obtained by ANOM illustrate qualitatively how the short-circuit impedance is sensitive to the variations of design parameters. On the contrary, the purpose of ANOVA is to determine the relative importance of the various design variables.

The main objective of ANOVA is to extract the relative contribution of each factor to the total variation of the objective function. To determine the sensitivity of the objective function $F$ to the design variables variation, ANOVA computes the sum of squares (SS) according to the following equation [12]

$$SS_p = \sum_{i=1}^{5} (m_p(F_i) - m(F))^2$$

(47)

where $SS_p$ is the sum of squares of parameter $P$, $m_p(F_i)$ is the mean value of the objective function for the $i$ level of parameter $P$ and $m(F)$ is the average of all experiments. The sum of squares of other parameters can be calculated similarly. Comparison of the $SS_p$ with $SS$ of other parameters shows the relative importance of parameter with respect to other parameters. Table 5 lists the $SS$ of parameters $x_1$, $x_2$, $x_3$, $x_1x_2$, $x_1x_3$ and $x_2x_3$, along with the respective coefficient estimate of each parameter to the objective function (intercept corresponds to the constant coefficient of $F$). According to the results of Table 5, the analytical equation for the approximation of the objective function $F_{	ext{fitted}}$ (response function) is

$$F_{\text{fitted}} = 0.2740\bar{x}_1 - 0.1623\bar{x}_2 - 0.7298\bar{x}_3 - 0.9511\bar{x}_1 \times \bar{x}_2 + 0.8293\bar{x}_2 \times \bar{x}_3 + 0.8862\bar{x}_2 \times \bar{x}_3 + 1.2763$$

(48)

where $\bar{x}_1$, $\bar{x}_2$ and $\bar{x}_3$ are the normalised values of the variables $x_1$, $x_2$ and $x_3$, respectively, yielded by:

$$\bar{x}_1 = \frac{2}{x_1 \max - x_1 \min} x_1 - \frac{x_1 \max + x_1 \min}{2}$$

(49)

$$\bar{x}_2 = \frac{2}{x_2 \max - x_2 \min} x_2 - \frac{x_2 \max + x_2 \min}{2}$$

(50)

$$\bar{x}_3 = \frac{2}{x_3 \max - x_3 \min} x_3 - \frac{x_3 \max + x_3 \min}{2}$$

(51)

This function is used as objective function input in the optimisation algorithms of the next section.

Fig. 10 presents the ANOM effect plots of factors $x_1$, $x_2$ and $x_3$ on $F_{\text{mean}}$, that is the mean value of $F_{\text{FEM}}$ (mean value of the two numerical experiments corresponding to low and high mesh density), $F_{\text{standard}}$, that is the standard deviation of $F$ and the signal-to-noise ratio (SNR). The SNR corresponds to ‘larger-the-better’, that is SNR = 10log([square of expected value of the given data]/(variance of the given data)). According to Fig. 10, the factor influencing the most the objective function is $x_3$ (this can also be observed by comparison of the coefficients of Table 5). This can be attributed to the fact that, since $x_3$ is the distance between LV1 and LV2 windings, it determines the variation of $U_{\text{LV}_1 \rightarrow \text{LV}_2} = U_3$, which corresponds to the largest value among the three short-circuit impedance components, thus influencing the most the objective function. Increase in $x_3$ results to decrease of $F_{\text{mean}}$ and $F_{\text{standard}}$ and increase of SNR. Since the goal is to minimise $F$ and maximise SNR, larger values of this parameter are therefore desirable. However, owing to the interaction with the rest of the parameters of Table 5, the choice of the optimal value is not straightforward.

The Pareto plot allows the detection of the factor and interaction effects which are most important to the process or design optimisation study one has to deal with. It

Table 4 Table of numerical experiments for the 1000 kVA transformer (high mesh density)

<table>
<thead>
<tr>
<th>Numerical $x_i$, mm</th>
<th>mm $x_2$, mm $x_3$, mm $U_i$, % $U_2$, % $U_3$, % $F_{\text{FEM}}$, %</th>
<th>experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8 5 8 7.53 6.11 9.28 0.80</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>12 5 7 7.4 5.98 9.22 0.81</td>
<td></td>
</tr>
<tr>
<td>3</td>
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<td></td>
</tr>
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<td></td>
</tr>
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<td></td>
</tr>
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<td>11</td>
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<td></td>
</tr>
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<td></td>
</tr>
<tr>
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</tr>
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<td></td>
</tr>
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<td>15</td>
<td>8 3 6 8.38 6.83 13.44 3.73</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>10 4 9 7.45 6.18 9.11 0.94</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>12 7 9 7.73 6.25 9.72 0.65</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>10 3 8 7.52 6.12 9.02 1.04</td>
<td></td>
</tr>
<tr>
<td>19</td>
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<td>25</td>
<td>9 7 6 8.14 6.44 10.28 1.07</td>
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Table 5 ANOVA results for the 1000 kVA transformer

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<th>Coefficient</th>
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<td>7.4178</td>
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<tr>
<td>$x_2$</td>
<td>6.2355</td>
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<tr>
<td>$x_3$</td>
<td>2.6493</td>
<td>-0.7298</td>
</tr>
<tr>
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<td>8.0314</td>
<td>-0.9511</td>
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<tr>
<td>$x_1x_3$</td>
<td>8.0314</td>
<td>0.8293</td>
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<td>$x_2x_3$</td>
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<td>0.8862</td>
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<tr>
<td>intercept</td>
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<td></td>
</tr>
</tbody>
</table>
displays the absolute values of the effects, and draws a reference line on the chart. Any effect that extends past this reference line is potentially important [11]. Fig. 11 presents the Pareto charts for $F_{\text{mean}}$, $F_{\text{standard}}$ and SNR. According to Fig. 11, while parameters $x_1$, $x_1x_2$, $x_1x_3$ and $x_2x_3$ are significant for the variation of $F_{\text{mean}}$, only $x_1x_2$ is significant for the variation of $F_{\text{standard}}$. On the contrary, none of the parameters of Table 5 is significant for the variation of SNR.

4.2.3 Verification: In order to verify the response function yielded by the ANOVA of the previous sections, the value of $F$ was calculated with the use of (48) (yielding the fitted values $F_{\text{fitted}}$) and the respective results were compared with

Fig. 10 ANOM plots for main factor effects

a $x_1$, $x_2$, $x_3$ effect on $F_{\text{mean}}$
b $x_1x_2$, $x_1x_3$ effect on $F_{\text{mean}}$
c $x_1$, $x_2$, $x_3$ effect on $F_{\text{standard}}$
d $x_1x_2$, $x_1x_3$, $x_2x_3$ effect on $F_{\text{standard}}$
e $x_1$, $x_2$, $x_3$ effect on SNR
f $x_1x_2$, $x_1x_3$, $x_2x_3$ effect on SNR

Fig. 11 Pareto charts for

a $F_{\text{mean}}$
b $F_{\text{standard}}$
c SNR
the actual values provided by the numerical experiments. The results of the comparison are listed in Table 6. Table 6 also includes the standard deviation of $F$ and the SNR at the considered $x_1$, $x_2$ and $x_3$.

The residual and standardised residual in Table 6 are given by

\[
\text{Residual} = F_{\text{mean}} - F_{\text{fitted}} \tag{52}
\]

\[
\text{Standardised residual} = \frac{F_{\text{mean}} - F_{\text{fitted}}}{SE} \tag{53}
\]

where SE is the standard error of the model, equal to 0.668.

According to Table 6, and the respective probability density function illustrated in Fig. 12, most of the absolute values of the standardised residuals are lower than 1 and only one exceeds the value of 2, indicating that the response function provided by the Taguchi method fits well the data of the numerical experiments.

### 4.3 Optimisation process

A combined optimisation technique (comprising a first stochastic step for global search and a second deterministic one for fast and effective local minimisation) is proposed. More specifically, the optimisation process has been divided into two cycles: in the first cycle, the stochastic optimisation algorithms of Table 7 are used to derive three different optimum values for the design vector. Three stochastic methods have been selected for the optimisation process, with wide applicability in electromagnetic problems: (i) Genetic Algorithms (GA), which have seen increased usage in the transformer design area over the last few years [14, 15]; (ii) Monte Carlo (MC), a stochastic method basically used for problems faced with uncertainty in many directions or too complex for analytical solution, therefore effective for application to electric machines and transformer design problems [16]; (iii) Simulated Annealing (SA), which has also proved to be quite effective in optimisation techniques for this class of problems [17].

In the second cycle, the best optimum value is used as start point for a deterministic optimisation algorithm (Pattern Search), resulting to the final global optimum. This process is also depicted in the flowchart of Fig. 13.

#### 4.3.1 First optimisation cycle: Genetic Algorithm

The GA metaheuristic is traditionally applied to discrete

<table>
<thead>
<tr>
<th>Numerical experiment</th>
<th>$F$(-)-low mesh density</th>
<th>$F$(-)-high mesh density</th>
<th>$F_{\text{mean}}, %$</th>
<th>$F_{\text{fitted}}, %$</th>
<th>Residual, %</th>
<th>Standardised residual, %</th>
<th>$F_{\text{standard}}, %$</th>
<th>SNR</th>
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</thead>
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<tr>
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<tr>
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<td>0.80</td>
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<td>0.69</td>
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<td>0.79</td>
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<td>0.24</td>
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<td>−0.14</td>
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<td>0.19</td>
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<td>8</td>
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<td>0.70</td>
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<td>−0.33</td>
<td>−0.50</td>
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<tr>
<td>9</td>
<td>1.08</td>
<td>0.81</td>
<td>0.94</td>
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<td>0.50</td>
<td>0.04</td>
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<td>−1.37</td>
<td>0.07</td>
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<td>0.91</td>
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<td>0.07</td>
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<td>0.25</td>
<td>0.37</td>
<td>1.25</td>
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</table>

Fig. 12 Standardised Residual Histogram for the ANOVA results of Table 5
Simulated Annealing is a Monte Carlo-based method employing more advanced stochastic search techniques to reach the minimum of the objective function. The idea is to create a sequence of points that linear constraints and bounds are satisfied and that converge to a global optimum of the objective function as the number of generations (iterations) increases. The limit for the fitness function is set to 0.5, while the maximum number of generations (iterations) is equal to 500.

Monte Carlo: The Monte Carlo optimisation process generates a random set of values for the design vector from within the permissible range and computes the respective value of the objective function. This process is continued until the stopping criterion is met (i.e. the objective function value is less than the objective limit defined by the user) or until the maximum number of iterations is reached (in which case, the design vector corresponding to the minimum value of the objective function is selected). This process can be time consuming, because a large number of design vectors is generally needed to cover the design space. The objective limit for $F_{\text{fitted}}$ in our case was set equal to 0.5 (in the first optimisation cycle, a large threshold is selected, in order to ensure convergence of the method) and the maximum number of iterations equal to 500 (same values as GA).

Simulated Annealing: Simulated Annealing is a Monte Carlo-based method employing more advanced stochastic search techniques to reach the minimum of the objective function. The idea is to create a sequence of points $X_1, X_2, \ldots$ that are approximately distributed according to probability density functions $p_1(x), p_2(x), \ldots$ with $p(x) \propto p(x)^{1/T}$, where $T_1, T_2, \ldots$ is a sequence of temperatures (known as the annealing schedule) that decreases to zero. If each $X_i$ are sampled exactly from $p(x)^{1/T}$, then $X_i$ would converge to a global optimum of the objective function as $T_i \to 0$. However, in practice, sampling is approximate and convergence to a global maximum is not assured [18].

In our case, Boltzmann function was used as the annealing function, that is the function used to generate new points at each iteration. The exponential temperature update was selected, yielding a temperature decrease equal to 0.95 per iteration. The objective limit for $F_{\text{fitted}}$ was set equal to 0.5 and the maximum number of iterations equal to 500 (same values as GA and MC).

Results: The results of the first cycle of the optimisation process are compared in Table 7. Owing to the stochastic nature of the algorithms, converging to different optimum designs for different executions, the mean value of the design vector and the optimum objective function value of ten runs of each algorithm is included in Table 7. Table 7 presents the value $F_{\text{fitted}}$ of the objective function as well as the optimal design vector. In order to verify the results, $F$
was also calculated by the use of the FEM model of high mesh density of Section 3, yielding the value $F_{\text{FEM}}$ of Table 7. Moreover, the percentage deviation $dU_1$, $dU_2$ and $dU_3$ of each short-circuit component (calculated by the FEM model) from the specified value $U_{\text{spec}}$, $U_{\text{spec}}$ and $U_{\text{spec}}$, respectively, expressed as percentage of the specified value, as well as the average value of these three deviations ($\overline{dU}$), is included in Table 7. Table 7 also includes the average number of iterations (based on the iterations of the ten runs).

GA requires the smaller number of iterations, and produces the optimal solution $F_{\text{GA}} = 0.497$ for $(x_1^{\text{GA}}, x_2^{\text{GA}}, x_3^{\text{GA}}) = (11.826, 6.604, 7.452)$, while the respective value calculated by FEM is $F_{\text{FEM}} = 0.677$. The deviations of $U_1$, $U_2$ and $U_3$ from their specified values (as computed by the FEM model) are equal to 3.958, 1.267 and 6.090%, respectively, yielding a mean deviation $\overline{dU}$ equal to 3.772%.

MC requires the larger number of iterations, as expected, and produces the optimal solution $F_{\text{MC}} = 0.399$ for $(x_1^{\text{MC}}, x_2^{\text{MC}}, x_3^{\text{MC}}) = (11.636, 7.000, 7.618)$, while the respective value calculated by FEM is $F_{\text{FEM}} = 0.714$. The deviations of $U_1$, $U_2$ and $U_3$ from their specified values (as computed by the FEM model) are equal to 3.681, 2.267 and 6.490%, respectively, yielding a mean deviation $\overline{dU}$ equal to 4.146%.

SA requires less iterations than MC but produces the worst optimal solution. More specifically, $F_{\text{SA}} = 0.621$ for $(x_1^{\text{SA}}, x_2^{\text{SA}}, x_3^{\text{SA}}) = (8.222, 6.345, 9.990)$, while the respective value calculated by FEM is $F_{\text{FEM}} = 0.698$. It must be noted that $F_{\text{SA}}$ is larger than the objective limit of 0.5 that is used as a stopping criterion for the algorithm, because it corresponds to the mean value of 10 executions of the algorithm, and in some of these executions the stopping criterion was not met, so the respective optimal solution was larger than the objective limit. The deviations of $U_1$, $U_2$ and $U_3$ from their specified values (as computed by the FEM model) are equal to 5.861, 3.350 and 5.190%, respectively, yielding a mean deviation $\overline{dU}$ equal to 4.800%.

In order to choose the best optimal solution, the values of $F_{\text{FEM}}$ of the three above solutions are compared (instead of $F_{\text{fitness}}$ since they are yielded by the more accurate FEM model than the response function of the regression model of Section 4.2.2). Among the three optimal values, $F_{\text{GA}}$ is the smallest one, so the corresponding design vector $(x_1^{\text{best}}, x_2^{\text{best}}, x_3^{\text{best}}) = (x_1^{\text{GA}}, x_2^{\text{GA}}, x_3^{\text{GA}})$ is used as the initial design vector $(x_1^0, x_2^0, x_3^0)$ in the second optimisation cycle, employing the Pattern Search algorithm (unlike the methods of the first cycle, that do not require an initial vector provided by the user).

### 4.3.2 Second optimisation cycle: Pattern Search

Pattern Search is a non-gradient optimisation method, where the search direction is cycled through the number of $n$ variables in sequence and then the $n + 1$ search direction is assembled as a linear combination of the previous $n$ search directions. In this optimisation cycle the objective limit is stricter (i.e. smaller than the one applied in the algorithms of the first optimisation cycle) and equal to 0.25.

Table 8 lists the results of this optimisation cycle, yielding a solution that decreases the objective function value (as calculated by the FEM model) from 0.677 (the value obtained by GA) to 0.349. Moreover, the respective $\overline{dU}$ is decreased from 3.772 to 2.171%. The number of iterations is significantly smaller than the respective number of the stochastic optimisation algorithms of the first cycle, as expected.

The results of the optimised design are compared with that of the real transformer (where the design was carried out by conventional analytical methods) described in Section 2 (which has been used for the measurements of Table 2 in Section 3.5). In the initial design, the measured values of $U_1$, $U_2$ and $U_3$ result to objective function $F$ equal to 0.56 (while the objective function value yielded by the proposed method is equal to 0.349, i.e. a decrease of 37.68% is achieved by the proposed method). Moreover, in the case of the initial transformer $dU_1 = 6.527\%$, $dU_2 = 2.800\%$ and $dU_3 = 2.171\%$, yielding $\overline{dU} = 3.776\%$ (while in the case of the optimised design $\overline{dU} = 2.171\%$, i.e. a decrease of 42.51% is achieved by the proposed method).

## 5 Conclusions

In the present study, a multi-objective design optimisation for double primary and secondary winding transformers was presented, with the aim to minimise the deviation between the prescribed and designed short-circuit impedance values. Finite element analysis was used to evaluate the objective function at different experiments and the results were analysed by the Taguchi method for the derivation of the objective function. The developed FEM model is able to achieve the maximum accuracy at the minimum computational cost, therefore it is particularly suitable for use with DOE. The Taguchi OA approach with noise factors yielded an analytical expression of the variation of short-circuit impedance components deviation from the specified values with respect to the distance between the four transformer windings. Finally, a combined stochastic-deterministic optimisation process was carried out, using this analytical expression, resulting to a better optimum value than the one yielded separately by stochastic algorithms.

According to the analysis presented in the study, the parameter affecting the most the objective function is the vertical distance between the HV and LV windings. Among the three stochastic optimisation algorithms that were used to optimise the design variables, GA produced the better solution in terms of objective function value and deviation of each short-circuit component from the specified value.

The second optimisation cycle, using PS and the initial design vector corresponding to the GA solution yielded a final optimum that decreased the objective function value from 0.677 (the value obtained by GA) to 0.349. Moreover, the respective $\overline{dU}$ is decreased from 3.772 to 2.171%. The final optimal design was compared with the initial transformer design, carried out by conventional analytical methods. It was found that the optimised design achieves a decrease of 37.68% at the objective function value and 42.51% at the mean deviation of the short-circuit impedance components from the specified values.

### Table 8 Pattern search results (second cycle of optimisation) for the 1000 kVA transformer

<table>
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<tr>
<th>$x_1$, mm</th>
<th>$x_2$, mm</th>
<th>$x_3$, mm</th>
<th>$F_{\text{fitness}}$, %</th>
<th>$F_{\text{FEM}}$, %</th>
<th>$dU_1$, %</th>
<th>$dU_2$, %</th>
<th>$dU_3$, %</th>
<th>$\overline{dU}$, %</th>
<th>Niter</th>
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<tr>
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<td>9.994</td>
<td>0.100</td>
<td>0.349</td>
<td>3.750</td>
<td>0.583</td>
<td>2.180</td>
<td>2.171</td>
<td>9</td>
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6 References

7. IEC 60076–1, Power Transformers—Part 1: General 2000